

## Recitation Worksheet (Optional Extra Practice)

Name:

MyID:

### Textbook:

Chemistry & Chemical Reactivity

by John C. Kotz, Paul M. Treichel, John R. Townsend, David Treichel

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### Instructions:

- This recitation worksheet is optional extra practice for 17.3, 19.1
- You **do not** need to submit it to Gradescope.
- The answer key has been posted with this worksheet to eLC.
- A periodic table and formula sheet are attached to the end of this worksheet.

1. Calculate the molar solubility in mol/L of:



$4.5 \times 10^{-5} \text{ M}$

|             | $[\text{Mg}^{2+}]$ | $[\text{AsO}_4^{3-}]$ |
|-------------|--------------------|-----------------------|
| Initial     | 0                  | 0                     |
| Change      | +3s                | +2s                   |
| Equilibrium | 3s                 | 2s                    |

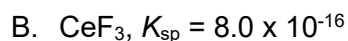
②  $K_{sp} = [\text{Mg}^{2+}]^3 [\text{AsO}_4^{3-}]^2$   
 $= [3s]^3 [2s]^2$   
 $= 108s^5$

$\therefore 2.1 \times 10^{-20} = 108s^5$

$\therefore s = \sqrt[5]{\frac{2.1 \times 10^{-20}}{108}}$

$= 4.547357596 \times 10^{-5} \text{ M}$

$\approx 4.5 \times 10^{-5}$



$7.4 \times 10^{-5} \text{ M}$



|             | $[\text{Ce}^{3+}]$ | $[\text{F}^-]$ |
|-------------|--------------------|----------------|
| Initial     | 0                  | 0              |
| Change      | +s                 | +3s            |
| Equilibrium | s                  | 3s             |

②  $K_{sp} = [\text{Ce}^{3+}] [\text{F}^-]^3$   
 $8.0 \times 10^{-16} = [s] [3s]^3$   
 $= 27s^4$

$\therefore s = \sqrt[4]{\frac{8.0 \times 10^{-16}}{27}}$

$= 7.377879463 \times 10^{-5} \text{ M}$

$\approx 7.4 \times 10^{-5} \text{ M}$

3. Which compound has the highest molar solubility?

**E**

- A. AgCN,  $K_{sp} = 5.97 \times 10^{-17}$   
 B. PbSO<sub>4</sub>,  $K_{sp} = 1.82 \times 10^{-8}$   
 C. PbS,  $K_{sp} = 9.04 \times 10^{-29}$   
 D. NiS,  $K_{sp} = 3.00 \times 10^{-20}$   
 E. MgCO<sub>3</sub>,  $K_{sp} = 6.82 \times 10^{-6}$

All ionic compounds have the same dissociation stoichiometry  $\therefore$  they can be ranked by solubility using  $K_{sp}$   
 highest  $K_{sp} \therefore$  highest solubility



4. Calculate  $K_{sp}$  given the molar solubilities of the following compounds:

A. Barium phosphate, molar solubility  $8.89 \times 10^{-9}$  M

$6.00 \times 10^{-39}$



|             | $[Ba^{2+}]$ | $[PO_4^{3-}]$ |
|-------------|-------------|---------------|
| Initial     | 0           | 0             |
| Change      | $+3s$       | $+2s$         |
| Equilibrium | $3s$        | $2s$          |

②  $K_{sp} = [Ba^{2+}]^3 [PO_4^{3-}]^2$   
 $= [3s]^3 [2s]^2$   
 $= 108s^5$

$= 108 (8.89 \times 10^{-9})^5$   
 $= 5.99697945 \times 10^{-39}$   
 $\approx 6.00 \times 10^{-39}$

B. Ag<sub>2</sub>S, molar solubility  $1.26 \times 10^{-16}$  M

$8.00 \times 10^{-48}$



|             | $[Ag^+]$ | $[S^{2-}]$ |
|-------------|----------|------------|
| Initial     | 0        | 0          |
| Change      | $+2s$    | $+s$       |
| Equilibrium | $2s$     | $s$        |

②  $K_{sp} = [Ag^+]^2 [S^{2-}]$   
 $= [2s]^2 [s]$   
 $= 4s^3$

$\therefore K_{sp} = 4 (1.26 \times 10^{-16})^3$   
 $= 8.001504 \times 10^{-48}$   
 $\approx 8.00 \times 10^{-48}$

5. There are some data that suggest that zinc lozenges can significantly shorten the duration of a cold. If the solubility of zinc acetate,  $\text{Zn}(\text{CH}_3\text{COO})_2$ , is 43.0 g/L, what is the solubility product,  $K_{sp}$  of this compound?

① convert 43.0 g/L to mol/L

$$\frac{43.0 \text{ g Zn}(\text{CH}_3\text{COO})_2}{1 \text{ L}} \times \frac{1 \text{ mol Zn}(\text{CH}_3\text{COO})_2}{183.48 \text{ g Zn}(\text{CH}_3\text{COO})_2} = 0.2343579682 \text{ M}$$

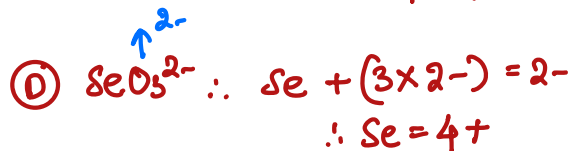
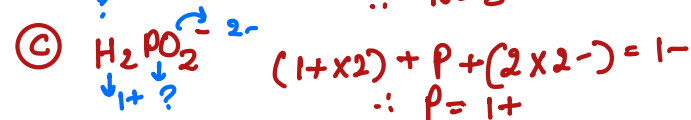
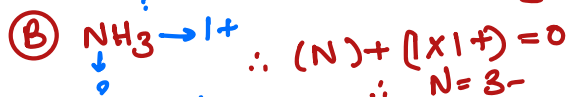
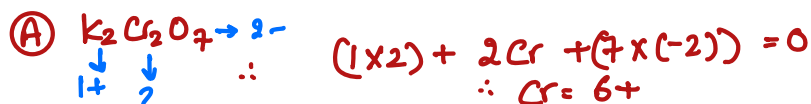
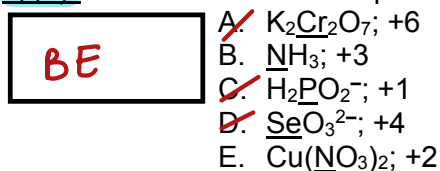
$5.15 \times 10^{-2}$   
or 0.0515



|             | $[\text{Zn}^{2+}]$ | $[\text{CH}_3\text{COO}^-]$ |
|-------------|--------------------|-----------------------------|
| Initial     | 0                  | 0                           |
| Change      | +S                 | +2S                         |
| Equilibrium | S                  | 2S                          |

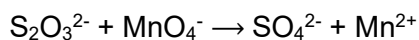
③  $K_{sp} = [\text{Zn}^{2+}][\text{CH}_3\text{COO}^-]^2$   
 $[S][2S]^2 = 4S^3 = 4(0.2343579682)^3 = 0.05148718689 \approx 0.0515 \text{ or } 5.15 \times 10^{-2}$

6. Which assignment of oxidation number is **INCORRECT** for the underlined element? Select all that apply. Insert letters without spaces in the answer box, example **ABCD**.

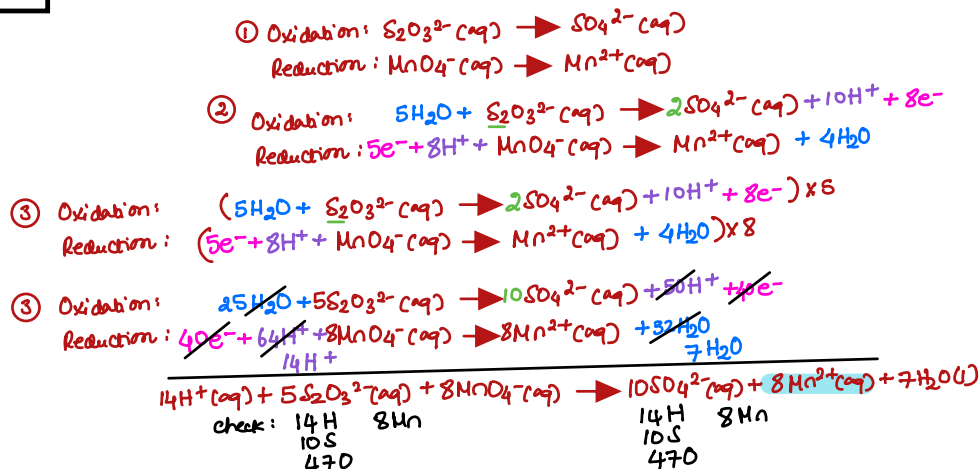


7. What is the coefficient of  $\text{Mn}^{2+}$  if the reaction below occurs in acidic solution?

C



- A. 5  
B. 7  
C. 8  
D. 10  
E. 14



8. What is the oxidation half-reaction in the chemical reaction  $\text{Cr}_2\text{O}_7^{2-}(\text{aq}) + \text{Cl}^-(\text{aq}) \rightarrow \text{Cr}^{3+}(\text{aq}) + \text{Cl}_2(\text{aq})$  balanced in acidic medium?

A

- A.  $2\text{Cl}^-(\text{aq}) \rightarrow \text{Cl}_2(\text{aq}) + 2\text{e}^-$   
B.  $\text{Cl}_2(\text{aq}) + 2\text{e}^- \rightarrow 2\text{Cl}^-(\text{aq})$   
C.  $\text{Cr}_2\text{O}_7^{2-}(\text{aq}) + 14\text{H}^+(\text{aq}) + 6\text{e}^- \rightarrow 2\text{Cr}^{3+}(\text{aq}) + 7\text{H}_2\text{O}(\text{l})$   
D.  $\text{Cr}_2\text{O}_7^{2-}(\text{aq}) + 14\text{H}^+(\text{aq}) \rightarrow 2\text{Cr}^{3+}(\text{aq}) + 7\text{H}_2\text{O}(\text{l}) + 6\text{e}^-$

Oxidation half-reaction:

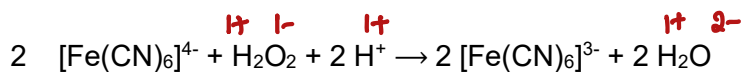


Reduction half-reaction:



9. What is the reducing agent in the following reaction between hexacyanoferrate (II) complex and hydrogen peroxide in acidic solution?

A

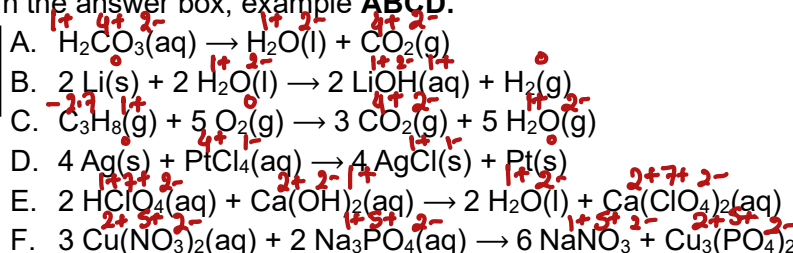


Reducing agent is oxidized (oxidation state of  $[\text{Fe}(\text{CN})_6]$  decreases)

- A.  $[\text{Fe}(\text{CN})_6]^{4-}$   
B.  $\text{H}_2\text{O}_2$   
C.  $\text{H}^+$   
D.  $[\text{Fe}(\text{CN})_6]^{3-}$   
E.  $\text{H}_2\text{O}$

10. Which of the following equations are an oxidation-reduction reaction? Select all that apply. Insert letters without spaces in the answer box, example ABCD.

BCD



Redox reactions:

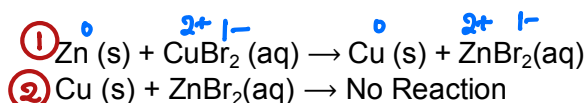
- 1) Single replacement (single displacement)
- 2) Combustion reactions

Reactions that are not redox:

- 1) Double replacement
- 2) Acid-base reactions
- 3) Decomposition reactions

11. Given the following laboratory observation, which of the following statements is FALSE?

C



- ~~A.~~ Zn is a stronger reducing agent than Cu. **True**  
~~B.~~  $\text{Cu}^{2+}$  is a stronger oxidizing agent than  $\text{Zn}^{2+}$ . **True**  
 C. Cu is a stronger reducing agent than Zn.  
~~D.~~ The fact that Cu doesn't react with  $\text{ZnBr}_2$  proves that copper attracts electrons more than does Zn.  
~~E.~~ None of the above.

From reaction 1: In the single replacement reaction Zn replaces  $\text{Cu}^{2+}$  in  $\text{CuBr}_2$  & gets oxidized  $\therefore \text{Zn}^0$  is a stronger reducing agent than Cu (also because reaction 2 does not occur)

12. The solubility of a salt  $\text{MX}_2$  with a molar mass of 114 g/mol is 3.42 g/L. Calculate  $K_{\text{sp}}$ .

B

- A.  $2.70 \times 10^{-5}$   
 B.  $1.08 \times 10^{-4}$   
 C.  $9.00 \times 10^{-4}$   
 D.  $2.25 \times 10^{-4}$   
 E.  $6.75 \times 10^{-8}$



|             | $[\text{M}^{2+}]$ | $[\text{X}^{-}]$ |
|-------------|-------------------|------------------|
| Initial     | 0                 | 0                |
| Change      | +S                | +2S              |
| Equilibrium | S                 | 2S               |

② Convert  $\frac{3.42 \text{ g MX}_2}{\text{L}} \times \frac{1 \text{ mol MX}_2}{114 \text{ g MX}_2} = \frac{0.030 \text{ mol}}{\text{L}} = S$   
 $\therefore K_{\text{sp}} = 4(0.030)^3 = 1.08 \times 10^{-4}$

$K_{\text{sp}} = [\text{M}^{2+}][\text{X}^{-}]^2$   
 $\therefore K_{\text{sp}} = [S][2S]^2 = 4S^3$   
 (brackets indicate conc. in  $\frac{\text{mol}}{\text{L}}$ )

13. Select a statement that best describes the oxidation process.

D

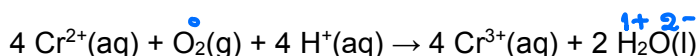
- some elements*
- A. In the oxidation process ~~all elements~~ change oxidation state. *some elements*
- B. In the oxidation process ~~all elements~~ experience an increase in oxidation state.
- C. In the oxidation process some elements change their oxidation state.
- D. In the oxidation process some elements experience oxidation state increase.
- E. In the oxidation process only oxygen increases its oxidation state.



*Other elements can also experience oxidation or an increase oxidation state*

14. Trace amounts of oxygen gas can be "scrubbed" from gases using the following reaction:

A



Which of the following statements is true regarding this reaction?

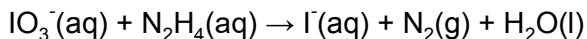
- A. Oxygen gas is reduced to water.
- B.  $\text{Cr}^{2+}(\text{aq})$  is the ~~oxidizing agent~~. *reducing agent because it gets oxidized to  $\text{Cr}^{3+}$*
- C.  $\text{O}_2(\text{g})$  is the ~~reducing agent~~. *oxidizing agent because it gets reduced to  $\text{O}^{2-}$*
- D. Electrons are transferred from  $\text{O}_2$  to  $\text{Cr}^{2+}$ .



*electrons are transferred from  $\text{Cr}^{2+}$  to  $\text{O}_2$*

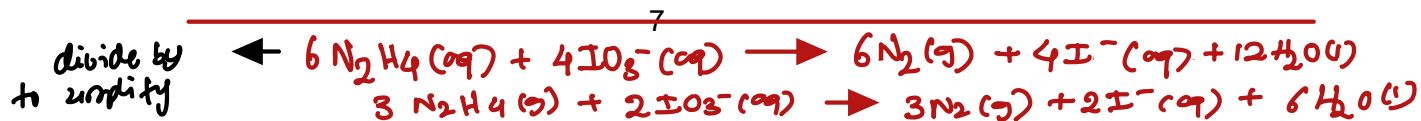
15. Balance the following equation for an oxidation-reduction reaction occurring in an acidic solution:

D



The sum of the coefficients is

- A. 8
- B. 25
- C. 18
- D. 16
- E. 32



## Formula Sheet

### Length

1 kilometer = 0.62137 mile  
1 inch = 2.54 centimeters (exactly)  
1 Ångstrom =  $1 \times 10^{-10}$  meter

### Energy

1 joule =  $1 \text{ kg} \cdot \text{m}^2 / \text{s}^2$   
1 calorie = 4.184 joules  
1 Calorie = 1 kilocalorie = 1000 calories  
1 L·atm = 101.325 joules

### Pressure

1 pascal =  $1 \text{ N} / \text{m}^2 = 1 \text{ kg} / \text{m} \cdot \text{s}^2$   
1 atmosphere = 101.325 kilopascals = 760 mm Hg = 760 torr = 14.70 lb/in<sup>2</sup>  
1 bar =  $1 \times 10^5$  Pa (exactly)

### Temperature

0 K = -273.15°C  
K = °C + 273.15  
°C = (5/9)(°F - 32)

### Mass

1 kg = 2.205 lbs

### Volume

1 mL =  $1 \text{ cm}^3 = 1 \text{ cc}$

### Constants

$c = 2.998 \times 10^8 \text{ m/sec}$   
 $h = 6.626 \times 10^{-34} \text{ J} \cdot \text{sec}^{-1}$   
 $R = 0.08206 \text{ L} \cdot \text{atm} / \text{mol} \cdot \text{K} = 8.314 \text{ J} / \text{mol} \cdot \text{K}$   
Specific heat of water = 4.184 J/g·K  
Mass of an electron:  $9.109 \times 10^{-31} \text{ kg}$   
Mass of a proton:  $1.673 \times 10^{-27} \text{ kg}$   
 $RH = 2.18 \times 10^{-18} \text{ J}$   
Specific heat of water = 4.184 J/g·K  
STP = 273.15 K and 1 atm  
Avogadro's number:  $6.022 \times 10^{23}$

### Equations

$d \text{ (density)} = m/V$   
 $P_1 V_1 = P_2 V_2$   
 $V_1/T_1 = V_2/T_2$   
 $P_1 V_1/n_1 T_1 = P_2 V_2/n_2 T_2$   
 $PV = nRT$   
 $(P + a(n^2/V^2)) \cdot (V - nb) = nRT$   
molar mass (M) =  $mRT/PV$   
density (d) =  $MP/RT$   
 $x_A = n_A/n_{\text{tot}} = P_A/P_{\text{tot}} = V_A/V_{\text{tot}}$   
 $P_{\text{tot}} = P_A + P_B + \dots$   
 $n_{\text{tot}} = n_A + n_B + \dots$   
$$\mu_{rms} = \sqrt{\frac{3RT}{M}}$$



$$\frac{\text{Rate of effusion A}}{\text{Rate of effusion B}} = \sqrt{\frac{MW_B}{MW_A}}$$

$$Q = C \times \Delta T = c_{\text{specific}} \times m \times \Delta T$$

$$Q = n \times \Delta H \text{ (kJ/mol)} = m \times \Delta H \text{ (kJ/g)}$$

$$w = -P\Delta V$$

$$\Delta E = q + w$$

$$\Delta H^\circ = \sum n\Delta H_f^\circ(\text{products}) - \sum n\Delta H_f^\circ(\text{reactants})$$

$$\Delta H^\circ = \sum n\Delta H^\circ(\text{bonds broken}) - \sum n\Delta H^\circ(\text{bonds formed})$$

$$E = h\nu$$

$$c = \lambda\nu$$

$$\lambda = h/mv$$

$$\Delta E = -2.18 \times 10^{-18} J \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

$$\ln\left(\frac{P_2}{P_1}\right) = \frac{\Delta H_{vap}}{R} \left( \frac{1}{T_1} - \frac{1}{T_2} \right)$$

$$C_g = kP_g$$

$$P_{\text{solution}} = P_{\text{solvent}} X_{\text{solvent}}$$

$$P_{\text{solution}} = \sum P_j = \sum P_j X_j$$

$$\Delta T_b = K_b m_i$$

$$\Delta T_f = K_f m_i$$

$$\pi = MRTi$$

### Thermodynamic and Electrochemistry

$$S = k_b \times \ln(W)$$

$$k_b = 1.381 \times 10^{-23} \text{ J/K}$$

$$\Delta S = q_{\text{rev}}/T$$

$$\Delta S_{\text{surr}} = q_{\text{surr}}/T = -q_{\text{rev}}/T$$

$$\Delta S_{\text{univ}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}}$$

$$\Delta S^\circ_{\text{rxn}} = \sum \nu S^\circ_{\text{products}} - \sum \nu S^\circ_{\text{reactants}}$$

$$\Delta H^\circ_{\text{rxn}} = \sum \nu H^\circ_{\text{products}} - \sum \nu H^\circ_{\text{reactants}}$$

$$\Delta G^\circ_{\text{rxn}} = \sum \nu G^\circ_{\text{products}} - \sum \nu G^\circ_{\text{reactants}}$$

$$\Delta G = \Delta H - T\Delta S$$

$$\Delta G = \Delta G^\circ + RT \cdot \ln Q$$

$$R = 8.314 \text{ J/mol.K}$$

$$\Delta G^\circ = -RT \cdot \ln K$$

$$\Delta G = -nFE_{\text{cell}}$$

$$F = 96485 \text{ J/(V}\cdot\text{mol e}^-)$$

$$E^\circ_{\text{cell}} = RT/nF \ln K$$

$$E^\circ_{\text{cell}} = (0.0257/n) \ln K = (0.0592/n) \log K$$

$$E_{\text{cell}} = E^\circ_{\text{cell}} - (RT/nF) \ln Q$$

$$E_{\text{cell}} = E^\circ_{\text{cell}} - (0.0257/n) \ln Q$$

$$\text{Electrolysis: } Q \text{ (total charge)} = I \times t = n \times F$$

### Integrated Rate Laws & half-life

$$\ln \frac{[A]}{[A]_0} = -kt$$

$$\frac{1}{[A]} = kt + \frac{1}{[A]_0}$$

$$[A] = -kt + [A]_0$$

$$t_{1/2} = \frac{[A]_0}{2k}$$

$$t_{1/2} = \frac{\ln 2}{k} = \frac{0.693}{k}$$

$$t_{1/2} = \frac{1}{k[A]_0}$$

$$\ln \frac{k_2}{k_1} = -\frac{E_a}{R} \left( \frac{1}{T_2} - \frac{1}{T_1} \right)$$

### **Equilibrium and Acid / Base**

$$K_p = K_c \times (RT)^{\Delta n}$$

$$\text{pH} = -\log[\text{H}_3\text{O}^+]$$

$$K_w = 1.0 \times 10^{-14} \text{ at } 25^\circ\text{C}$$

$$K_w = [\text{H}_3\text{O}^+] \times [\text{OH}^-]$$

$$K_w = K_a \times K_b$$

$$\text{p}K_a = -\log[K_a]$$

$$\text{Buffer: pH} = \text{p}K_a + \log \frac{[\text{A}^-]}{[\text{HA}]}$$

$$\ln \frac{K_2}{K_1} = \frac{\Delta H_{rxn}^\circ}{R} \left( \frac{1}{T_1} - \frac{1}{T_2} \right)$$

# Periodic Table of the Elements

|                    |  |                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |                    |  |                    |  |
|--------------------|--|--------------------|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--------------------|--|--------------------|--|
| 1                  |  |                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 2                  |  | 18                 |  |
| 1<br>H<br>1.01     |  | 2                  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | He<br>4.00         |  |                    |  |
| 3<br>Li<br>6.94    |  | 4<br>Be<br>9.01    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 5<br>B<br>10.81    |  | 6<br>C<br>12.01    |  |
| 11<br>Na<br>22.99  |  | 12<br>Mg<br>24.31  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 7<br>N<br>14.01    |  | 8<br>O<br>16.00    |  |
|                    |  |                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 9<br>F<br>19.00    |  | 10<br>Ne<br>20.18  |  |
|                    |  |                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 13<br>Al<br>26.98  |  | 14<br>Si<br>28.09  |  |
|                    |  |                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 15<br>P<br>30.97   |  | 16<br>S<br>32.06   |  |
|                    |  |                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 17<br>Cl<br>35.45  |  | 18<br>Ar<br>39.95  |  |
| 19<br>K<br>39.10   |  | 20<br>Ca<br>40.08  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 31<br>Ga<br>69.72  |  | 32<br>Ge<br>72.63  |  |
| 37<br>Rb<br>85.47  |  | 38<br>Sr<br>87.62  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 47<br>Ag<br>107.87 |  | 48<br>Cd<br>112.41 |  |
| 37<br>Cs<br>132.91 |  | 56<br>Ba<br>137.33 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 79<br>Au<br>196.97 |  | 80<br>Hg<br>200.59 |  |
| 87<br>Fr<br>[223]  |  | 88<br>Ra<br>[226]  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 111<br>Rg<br>[282] |  | 112<br>Cn<br>[285] |  |
|                    |  |                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 113<br>Nh<br>[286] |  | 114<br>Fl<br>[290] |  |
|                    |  |                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 115<br>Mc<br>[290] |  | 116<br>Lv<br>[293] |  |
|                    |  |                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 117<br>Ts<br>[294] |  | 118<br>Og<br>[294] |  |
|                    |  |                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |                    |  |                    |  |
|                    |  |                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |                    |  |                    |  |
|                    |  |                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |                    |  |                    |  |
|                    |  |                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |                    |  |                    |  |
|                    |  |                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |                    |  |                    |  |
|                    |  |                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |                    |  |                    |  |
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